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DESC 2023 - Seminar on

Differential Equations with Symbolic Computation

Seminar Program

Beihang University

Beijing, China, September 16-17, 2023

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The main objective of the seminar is to bring together foreign experts and Chinese researchers actively working on symbolic methods and techniques for differential equations to exchange ideas, views, and new results and to build up contacts for future cooperation. Your work on any aspect interrelating differential equations with symbolic computation will be considered for presentation at the seminar. Specific topics include:

- *Symbolic integration and solution of differential equations*
- *Differential algebra, differential algebraic geometry, and differential Galois theory*
- *Symbolic methods for qualitative analysis of differential equations and dynamical systems*
- *Applications of symbolic computation in differential equations*
- *Differential equations for symbolic-numeric computation*

Organizers:
Brigita Ferčec (University of Maribor, Slovenia)
Bo Huang (Beihang University, China)
Valery Romanovski (CAMTP and University of Maribor, Slovenia)
Dongming Wang (Beihang University, China)
Zhiming Zheng (Beihang University, China)

Local Arrangements:
Bo Huang (Beihang University, China)
Chenqi Mou (Beihang University, China)
Wei Niu (Beihang University, China)

Location: Ruxin Convention Center, Beihang University

Sponsors:

Beihang University

School of Mathematical Sciences, Institute of Artificial Intelligence

MOE Key Laboratory of Mathematics, Informatics and Behavioral Semantics

For more information, please see the website https://math-bo.github.io/DESC2023/

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Saturday September 16th, 2023

Time	
8:00	***Registration***
8:30 - 8:40	***Welcome***
8:40 - 9:30	**Session 1: Invited talk 1** **Chair**: Jibin Li **Maoan Han** - Bifurcation Theory of Limit Cycles by Higher Order Melnikov Functions and Applications
9:30 - 10:20	**Session 2: Invited talk 2** **Chair**: Zhaosheng Feng **Dongmei Xiao** - The Integrability and Global Dynamics of Kolmogorov Polynomial Differential Systems
10:20 - 10:50	***Break and Group Photo***
	Session 3: Chair: Chenqi Mou
10:50 - 11:20	**Yun Tian** - Small Limit Cycles in Some Trigonometric Systems
11:20 - 11:50	**Brigita Ferčec** - A Blow-up Method for Proving Integrability and Linearizability of Some Planar Polynomial Systems of Differential Equations
11:50 - 12:20	**Shaoshi Chen** - A Dynamical Aspect of Symbolic Summation
12:20 - 14:10	***Lunch Break***
14:10 - 15:00	**Session 4: Invited talk 3** **Chair**: Dongmei Xiao **Valery Romanovski** - Computation of Normal Forms of ODEs for Systems with Many Parameters
15:00 - 15:50	**Session 5: Invited talk 4** **Chair**: Valery Romanovski **Jaume Giné** - Characterization of Degenerate Centers by its Complex Separatrices
15:50 - 16:10	***Break***
	Session 6: Chair: Wei Niu
16:10 - 16:40	**Mateja Grašič** - Invariants and Reversibility in Polynomial Systems of ODEs
16:40 - 17:10	**Barbara Arcet** - Integrability and Linearizability of 3-dimensional Quadratic Systems
17:10 - 17:40	**Ruyong Feng** - Specialization of Linear Differential Equations
18:00 -	***Reception Dinner***

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Sunday September 17th, 2023

Time	
8:30 - 9:20	**Session 7: Invited talk 5** **Chair**: Maoan Han **Jibin Li** - Bifurcations of Zq-equivariant Vector Fields and Possible Configurations of Limit Cycles
9:20 - 10:10	**Session 8: Invited talk 6** **Chair**: Dongming Wang **Weinian Zhang** - Analysis of Enzyme-Catalyzed Reaction Model
10:10 - 10:30	***Break***
	Session 9: Chair: Brigita Ferčec
10:30 - 11:00	**Alexander Grin** - Construction of a Global Algebraic Poincaré - Bendixson Annulus for the Rayleigh Equation
11:00 - 11:30	**Hebai Chen** - Global Phase Portraits of Generalized Polynomial Liénard System with a Unique Equilibrium and Periodic Annulus
11:30 - 12:00	**Bo Huang** - Algebraic Analysis of Zero-Hopf Bifurcations of Polynomial Differential Systems
12:00 - 14:00	***Lunch Break***
14:00 - 14:50	**Session 10: Invited talk 7** **Chair**: Wentao Huang **Zhaosheng Feng** - Bounded Wave Solutions of KdV-Burgers-type Equations
14:50 - 15:40	**Session 11: Invited talk 8** **Chair**: Valery Romanovski **Wentao Huang** - Algorithms of Periodic Constants and Their Applications
15:40 - 16:00	***Break***
	Session 12: Chair: Bo Huang
16:00 - 16:30	**Xingwu Chen** - Persistent Center of Planar Differential Systems
16:30 - 17:00	**Feng Li** - Singular Value of Three-Multiple Nilpotent Singular Point
17:00 - 17:30	**Xiaoxian Tang** - Structural Attractiveness of Low-Dimensional Zero-One Networks
17:30 - 17:40	***Closing***

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Titles and Abstracts

Invited talk 1：

Bifurcation Theory of Limit Cycles by Higher Order Melnikov Functions and Applications

Maoan Han (Zhejiang Normal University, China)

Abstract: In this paper, we study Poincaré, Hopf and homoclinic bifurcations of limit cycles for planar near-Hamiltonian systems. Our main results establish Hopf and homoclinic bifurcation theories by higher order Melnikov functions, obtaining conditions on upper bounds and lower bounds of the number of limit cycles. As an application, we concern a cubic near-Hamiltonian system, and study Hopf and homoclinic bifurcation in detail.

Invited talk 2：

The Integrability and Global Dynamics of Kolmogorov Polynomial Differential Systems

Dongmei Xiao (Shanghai Jiaotong University, China)

Abstract: In this talk, we first introduce background of planar polynomial differential systems, then provide a link between the integrability of planar polynomial Kolmogorov differential systems and the intersection number of planar algebraic curves, and obtain algebraic and computational conditions for ruling out the existence of limit cycles. To answer if global dynamics of an integrable system can be completely determined by its local dynamics of all equilibrium points in Poincar\'e disc, we study the Kolmogorov systems with degree $n\le 3$. It is proved that the Kolmogorov systems with degree $n\le 3$ are integrable if the number of center-type equilibria or weak saddles of these systems in the interior of quadrants of real plane $\mathbb{R}^2$ reaches the maximum. For these integrable Kolmogorov systems, we give all topological classifications of its global dynamics, which is shown that the local dynamics of the integrable Kolmogorov systems with degree $n=2$ can completely determine its global dynamics, but the local dynamics of the integrable Kolmogorov systems with degree $n=3$ cannot completely determine its global dynamics. This is based on a joint work with Dr. Hongjin He.

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Invited talk 3：

Computation of Normal Forms of ODEs for Systems with Many Parameters

Valery Romanovski (CAMTP and University of Maribor, Slovenia)

Abstract: There are two ways to compute Poincaré-Dulac normal forms of systems of ODEs. Under the original approach used by Poincaré the normalizing transformation is explicitly computed. On each step, the normalizing procedure requires the substitution of a polynomial to a series. Under the other approach, a normal form is computed using Lie transformations. In this case, the changes of coordinates are performed as actions of certain infinitesimal generators. In both cases, on each step the homological equation is solved in the vector space of polynomial vector fields V^n_j where each component of the vector field is a homogeneous polynomial of degree j. We present the third way of computing normal forms of polynomial systems of ODEs where the coefficients of all terms are parameters. Although we use Lie transforms the homological equation is solved not in V^n_j but in the vector space of polynomial vector fields where each component is a homogeneous polynomial in the parameters of the system. It is shown that the space of the parameters is a kind of dual space and, the computation of normal forms can be performed in the space of parameters treated as the space of generalized vector fields. The approach provides a simple way to parallelize the normal form computations opening the way to compute normal forms up to higher order than under previously known two approaches. This is a joint work with Tatjana Petek.

Reference.

1. T. Petek and V. G. Romanovski, Computation of Normal Forms for Systems with Many Parameters, arXiv:2305.01739, 2023.

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Invited talk 4：

Characterization of Degenerate Centers by its Complex Separatrices

Jaume Giné (Universitat de Lleida, Spain)

Abstract: In this talk we deal with analytic families of real planar vector fields $\mathcal{X}_\lambda$ having a monodromic singularity at the origin for any $\lambda \in \Lambda \subset \mathbb{R}^p$ and depending analytically on the parameters λ. There naturally appears the so-called center-focus problem which consists in describing the partition of Λ induced by the centers and the foci at the origin. We give a characterization of the centers (degenerated or not) in terms of a specific integral of the cofactor associated to a real invariant analytic curve passing through the singularity, which always exists. Several consequences and applications are also stated. The work is in collaboration with Prof. Isaac A. García

Invited talk 5：

Bifurcations of Zq-equivariant Vector Fields and Possible Configurations of Limit Cycles

Jibin Li (Zhejiang Normal University, China)

Abstract: Like the first part of Hilbert's 16th problem where the distribution of ovals is to be considered, the distribution problem of limit cycles can also be very interesting. Coleman [1983] in his survey" Hilbert 16th problem: How Many Cycles?" stated that "For n>2 the maximal number of eyes is not known, nor is it known just which complex patterns of eyes within eyes, or eyes enclosing more than a single critical point, can exist." Here so-called "eye" means the limit cycle.

In order to obtain more limit cycles and various configuration patterns of their relative dispositions, we indicated that an efficient method is to perturb the symmetric Hamiltonian systems having maximal number of centers, i.e. to study the weakened Hilbert's 16th problem for the symmetric planar polynomial Hamiltonian systems, since bifurcation and symmetry are closely connected and symmetric systems play pivotal roles as a bifurcation

point in all planar Hamiltonian system class. To investigate perturbed Hamiltonian systems, we should first know the global behavior of unperturbed polynomial systems, namely, determine the global property for the families of real planar algebraic curves defined by the Hamiltonian functions. Then by using proper perturbation techniques, we shall obtain the global information of bifurcations for the perturbed nonintegrable systems.

In this talk, we introduce the method of detection functions posed by Li et al. [1985], to study the Zq-equivariant perturbed polynomial systems and the method of control parameters. We also present recent developments in this direction.

Invited talk 6:

Analysis of Enzyme-Catalyzed Reaction Model

Weinian Zhang (Sichuan University, China)

Abstract: In this talk we discuss a substrate-activator system, which depends on a cubic polynomial with such a complicated relation between its coefficients and the original parameters that the coordinates of equilibria or even the number of equilibria can hardly be determined in many cases. All found results on its qualitative properties and bifurcations are given indirectly for the artificial parameter s_*, a coordinate of a general equilibrium, and the analysis of its dynamics remains far from completion. Not following the common idea of computing eigenvalues at equilibria, we give a complete analysis of equilibria directly for those original parameters by using continuity, monotonicity and some techniques of inequality. For a global analysis we discuss its equilibria at infinity, one of which possesses degeneracy so high sometimes that neither the well-known normal sector method nor the blowing-up method can be used easily. Furthermore, overcoming those difficulties from not solving all coordinates of equilibria, we give a versal unfolding with its original parameters to the degenerate cases and present bifurcation curves of periodic orbits and homoclinic orbits explicitly.

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Invited talk 7：

Bounded Wave Solutions of KdV-Burgers-type Equations

Zhaosheng Feng (University of Texas Rio Grande Valley, USA)

Abstract: In this talk, we consider dynamics of a class of the KdV-Burgers-type systems by starting with Burgers-type equations, and then focus on the higher-order KdV-Burgers equation, a partial differential equation that occupies a prominent position in describing some physical processes in motion of turbulence and other unstable process systems. We limit our attention to various bounded wave solutions and their asymptotic behaviors.

Invited talk 8：

Algorithms of Periodic Constants and Their Applications

Wentao Huang (Guangxi Normal University, China)

Abstract: We introduce algorithms of periodic constants for two-dimensional, three-dimensional and four-dimensional differential systems. The applications of the above algorithms in the isochronous center and the critical period bifurcation of differential systems are given.

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Short talk 1：

Small Limit Cycles in Some Trigonometric Systems

Yun Tian (Shanghai Normal University, China)

Abstract: In this talk, we study the number of small limit cycles around a singular point in trigonometric systems. Our first result gives sharp upper bounds on the number of zeros of the associated first order Melnikov functions near h=0 for pendulum equations under small perturbations. Some further results give Hopf cyclcity at the origin for some trigonometric Liénard systems.

Short talk 2：

A Blow-up Method for Proving Integrability and Linearizability of Some Planar Polynomial Systems of Differential Equations

Brigita Ferčec (University of Maribor, Slovenia)

Abstract: I shall discuss an effective method for proving integrability and linearizability of the resonant saddles. The method is based on the use of a blow-up transformation and solving the recurrence differential equations using induction. Using this method some open integrability and linearizability problems for certain resonant saddles are solved. We also apply the method to nilpotent and degenerate monodromic singularities solving the center problem when the center conditions are algebraic.

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Short talk 3：

A Dynamical Aspect of Symbolic Summation

Shaoshi Chen (Academy of Mathematics and System Sciences, CAS, China)

Abstract: Symbolic Summation aims in developing symbolic algorithms for evaluating and manipulating sums in combinatorics. In this talk, we will present some dynamical results related to some classical algorithms in Symbolic Summation. We will classify all of possible hypergeometric terms that are stable under the application of Gosper's algorithm and prove that P-recursive sequences are eventually stable under the iterated summation.

Short talk 4：

Invariants and Reversibility in Polynomial Systems of ODEs

Mateja Grašič (University of Maribor, Slovenia)

Abstract: We first investigate the interconnection of invariants of certain group actions and time-reversibility of a class of two-dimensional polynomial systems with $1:-1$ resonant singularity at the origin. The time-reversibility is related to the Sibirsky subvariety of the center (integrability) variety and it is known that every time-reversible system has a local analytic first integral at the origin. We propose a new algorithm to obtain a generating set for the Sibirsky ideal of such polynomial systems and investigate some algebraic properties of this ideal. Then, we discuss a generalization of the concept of time-reversibility in the three dimensional case considering the systems with $1:\zeta:\zeta^2$ resonant singularity at the origin (where ζ is a primitive cubic root of unity) and study a connection of such reversibility with the invariants of some group actions in the space of parameters of the system.

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Short talk 5:

Integrability and Linearizability of 3-dimensional Quadratic Systems

Barbara Arcet (University of Maribor, Slovenia)

Abstract: This presentation focuses on the aspects of integrability and linearizability in the context of systems of three-dimensional quadratic differential equations. The discussion centers on systems that display symmetry with respect to the plane y = 0. Our research demonstrates that all integrable systems within this specific family possess at least one first integral of the Darboux type. Additionally, our investigation delves into the concept of linearizability. We examine the necessary conditions for linearizability and identify families of systems that satisfy these conditions. Through the application of various methods, we establish sufficient conditions to prove the linearizability of these systems across most families, with the exception of a particular case.

Short talk 6:

Specialization of Linear Differential Equations

Ruyong Feng (Academy of Mathematics and System Sciences, CAS, China)

Abstract: Let k be an algebraically closed field of characteristic zero and let B be a finitely generated k-algebra that is an integral domain. We consider the following linear differential equation $$L(y)=a_n(x) y^{(n)}+a_{n-1}(x) y^{(n-1)}+\dots + a_0(x) y=0,\,\, a_i\in B[x]$$ as a family of linear differential equations parametrized by the variety $X(k)={\rm Hom}_k(B,k)$. Precisely, applying $c\in X(k)$ to the coefficients of each $a_i(x)$, one obtains a specialized differential equation $$ L^c(y)=a_n^c(x) y^{(n)}+a_{n-1}^c(x) y^{(n-1)}+\dots+a_0^c(x)y=0.$$ It is natural to ask how the algebraic properties of solutions of $L^c(y)=0$ vary as c ranges over $X(k)$. For instance, one may ask for which $c\in X(k)$ $L^c(y)=0$ has a basis of Liouvillian solutions, assuming that $L(y)=0$ does not have such basis. We call the set of such c the exceptional set of $X(k)$. In this talk, generalizing a result of Hrushovski, we show that the exceptional set is indeed

"small" in an appropriate sense. As an application, we prove Matzat's conjecture in full generality: The absolute differential Galois group of a one-variable function field over k, equipped with a non-trivial k-derivation, is the free proalgebraic group on a set of cardinality $|k|$. This is joint work with Michael Wibmer from University of Leeds, UK.

Short talk 7:

Construction of a Global Algebraic Poincaré - Bendixson Annulus for the Rayleigh Equation

Alexander Grin (Yanka Kupala State University of Grodno, Belarus)

Abstract: We consider the Rayleigh equation $\ddot{x} + \lambda (\dot{x}^2/3-1)\dot{x} +x=0$ depending on the real parameter λ and construct a Poincar\'e - Bendixson annulus $\mathcal{A}_\lambda$ in the phase plane containing the unique limit cycle Γ_λ of the Rayleigh equation for all $\lambda>0$. The novelty of this annulus consists in the fact that its boundaries are algebraic curves depending on λ. The polynomial defining the interior boundary represents a special Dulac-Cherkas function for the Rayleigh equation which immediately implies that the Rayleigh equation has at most one limit cycle. The outer boundary is the diffeomorphic image of the corresponding boundary for the van der Pol equation. Additionally we present some equations which are linearly topologically equivalent to the Rayleigh equation and provide also for these equations global algebraic Poincar\'e - Bendixson annuli.

Short talk 8:

Global Phase Portraits of Generalized Polynomial Liénard System with a Unique Equilibrium and Periodic Annulus

Hebai Chen (Central South University, China)

Abstract: This talk aims to provide a sufficient and necessary condition of the generalized polynomial Liénard system with a unique equilibrium and periodic annulus.

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Short talk 9：

Algebraic Analysis of Zero-Hopf Bifurcations of Polynomial Differential Systems

Bo Huang (Beihang University, China)

Abstract: The averaging method is a good tool for studying limit cycles of differential systems. In this talk, we first present an efficient symbolic program using Maple for computing the averaged functions of any order for continuous differential systems. The program allows us to systematically analyze zero-Hopf bifurcations of polynomial differential systems using symbolic computation methods. Then we study the number of limit cycles that may bifurcate from an equilibrium of an autonomous system of differential equations. The system in question is assumed to be of dimension n, have a zero-Hopf equilibrium at the origin, and consist only of homogeneous terms of order m. Denote by $H_k(n,m)$ the maximum number of limit cycles of the system that can be detected by using the averaging method of order k. We prove that $H_1(n,m)\leq(m-1)\cdot m^{n-2}$ and $H_k(n,m)\leq(km)^{n-1}$ for generic $n\geq3$, $m\geq2$ and $k>1$. The exact numbers of $H_k(n,m)$ or tight bounds on the numbers are determined by computing the mixed volumes of some polynomial systems obtained from the averaged functions. A number of examples are presented to demonstrate the effectiveness of the proposed algorithmic approach. Part of the work was completed jointly with Prof. Dongming Wang.

Short talk 10：

Persistent Center of Planar Differential Systems

Xingwu Chen (Sichuan University, China)

Abstract: We talk about center, isochronous center problems and persistence of center under perturbations for planar differential systems. We give some conditions for persistent center and weakly persistent center of some special systems by symbolic computations and analysis methods.

Short talk 11：

Singular Value of Three-Multiple Nilpotent Singular Point

Feng Li (Linyi University, China)

Abstract: In this talk, singular value of three-multiple nilpotent singular point is defined and computation method is give. When the three-multiple nilpotent singular point is a monodromic singular point, singular value can be treated as focal values. When the three-multiple nilpotent singular point is a saddle, singular value can be treated as analytical saddle values. We unify the focus quantity with the saddle point quantity of three-multiple nilpotent singular point. As an application, a class of quintic system with a three-multiple nilpotent origin is studied, analytical center conditions and analytical integral conditions of saddle are obtained respectively. Moreover, limit cycles bifurcation from the origin or infinity are also discussed. This is a joint work with Professor Pei Yu and Yuanyuan Liu.

Short talk 12：

Structural Attractiveness of Low-Dimensional Zero-One Networks

Xiaoxian Tang (Beihang University, China)

Abstract: Zero-one networks play a key role in cell signaling. A reaction network is structural attractive if the network admits at most one stable steady state. We show that if the dimension of a zero-one network is strictly less than three, then the network is structural attractive. We further classify all one-dimensional and two dimensional zero-one networks according to if they admit a stable steady state or not.