Construction of a global algebraic Poincaré – Bendixson annulus for the Rayleigh equation

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We consider the Rayleigh equation $\ddot{x} + \lambda(\dot{x}^2/3 - 1)\dot{x} + x = 0$ depending on the real parameter λ and construct a Poincaré – Bendixson annulus $\mathcal{A}_\lambda$ in the phase plane containing the unique limit cycle Γ_λ of the Rayleigh equation for all $\lambda > 0$. The novelty of this annulus consists in the fact that its boundaries are algebraic curves depending on λ. The polynomial defining the interior boundary represents a special Dulac–Cherkas function for the Rayleigh equation which immediately implies that the Rayleigh equation has at most one limit cycle. The outer boundary is the diffeomorphic image of the corresponding boundary for the van der Pol equation. Additionally we present some equations which are linearly topologically equivalent to the Rayleigh equation and provide also for these equations global algebraic Poincaré–Bendixson annuli.