Bifurcations of Zq-equivariant Vector Fields and Possible Configurations of Limit Cycles

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Like the first part of Hilbert's 16th problem where the distribution of ovals is to be considered, the distribution problem of limit cycles can also be very interesting. Coleman [1983] in his survey "Hilbert 16th problem: How Many Cycles?" stated that "For $n > 2$ the maximal number of eyes is not known, nor is it known just which complex patterns of eyes within eyes, or eyes enclosing more than a single critical point, can exist." Here so-called "eye" means the limit cycle.

In order to obtain more limit cycles and various configuration patterns of their relative dispositions, we indicated that an efficient method is to perturb the symmetric Hamiltonian systems having maximal number of centers, i.e. to study the weakened Hilbert's 16th problem for the symmetric planar polynomial Hamiltonian systems, since bifurcation and symmetry are closely connected and symmetric systems play pivotal roles as a bifurcation point in all planar Hamiltonian system class. To investigate perturbed Hamiltonian systems, we should first know the global behavior of unperturbed polynomial systems, namely, determine the global property for the families of real planar algebraic curves defined by the Hamiltonian functions. Then by using proper perturbation techniques, we shall obtain the global information of bifurcations for the perturbed nonintegrable systems.

In this talk, we introduce the method of detection functions posed by Li et al. [1985], to study the Zq-equivariant perturbed polynomial systems and the method of control parameters. We also present recent developments in this direction.