Computation of normal forms of ODEs for systems with many parameters

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There are two ways to compute Poincaré-Dulac normal forms of systems of ODEs. Under the original approach used by Poincaré the normalizing transformation is explicitly computed. On each step, the normalizing procedure requires the substitution of a polynomial to a series. Under the other approach, a normal form is computed using Lie transformations. In this case, the changes of coordinates are performed as actions of certain infinitesimal generators. In both cases, on each step the homological equation is solved in the vector space of polynomial vector fields V_j^n where each component of the vector field is a homogeneous polynomial of degree j. We present the third way of computing normal forms of polynomial systems of ODEs where the coefficients of all terms are parameters. Although we use Lie transforms the homological equation is solved not in V_j^n but in the vector space of polynomial vector fields where each component is a homogeneous polynomial in the parameters of the system. It is shown that the space of the parameters is a kind of dual space and, the computation of normal forms can be performed in the space of parameters treated as the space of generalized vector fields. The approach provides a simple way to parallelize the normal form computations opening the way to compute normal forms up to higher order than under previously known two approaches.

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Reference.
1. T. Petek and V. G. Romanovski, Computation of Normal Forms for Systems with Many Parameters, arXiv:2305.01739, 2023.